Practice Questions for Final Exam - Math 1060Q - Fall 2014

Before anyone asks, the final exam *is* cumulative. It will consist of about 50% problems on exponential and logarithmic functions, 25% problems on trigonometric functions, and 25% problems on other material covered in the course. Most of the problems will look like WebAssign problems, but as always, there will be some problems that require you to know concepts, understand definitions, and/or apply material you've learned in new ways.

You can go over the midterm reviews for more problems from earlier sections. The problems below will give you extra practice on exponential and logarithmic functions. The final exam will include trigonometric identities on it as they were on the second midterm.

You of course should be familiar with the rules of exponents. For one thing, *please* don't mix up $x^{\frac{1}{2}}$ and x^{-2}!

1. Simplify $\sqrt{(x^4x^2)^{\frac{3}{4}}}$.
2. Simplify $(\frac{3}{x})^{-2}$.
3. Simplify $\frac{x^{-1}y^2}{y^2x^{-2}}$.
4. Simplify $(\frac{x^{-2}}{x^{-3}})^{-4}$.
5. Simplify $\sqrt{x^{\frac{1}{3}}}x^{\frac{2}{5}}$.
6. Show that $(x+y)^2$ is not the same as x^2+y^2.
7. Simplify $\frac{1}{\frac{1}{x^{-1}}}$.
8. Is $(x+y)^{-1}$ the same as $x^{-1}+y^{-1}$?
9. Show that $x^{\frac{1}{2}}$ and x^{-2} are not the same.

Now that you understand exponents, you can work on exponential functions. What are their domain? Range? What do their graphs look like?

10. What is the domain of the function $f(x) = 2^x$? What is its range? What does its graph look like? Can you identify 4 points on the graph?
11. What is the domain of the function $g(x) = (\frac{1}{e})^x$? Range? Graph? Identify 4 points?
12. Graph the function $h(x) = (.3)^{x-2} + 5$.
13. What is the inverse of the function $j(x) = 5e^x$?
14. What is the inverse of the function $k(x) = 2 \cdot 3^{4x+5} + 6$?
15. What is the inverse of the function $l(x) = 3\log_4(x-1) + 1$?
16. What is the domain of the function $m(x) = \log_2(x+2)$?
17. What is the domain of the function $n(x) = \frac{1}{\ln(x-1)}$?
18. What is the domain of the function $\log_3(x^2 - x)$?

Then we come to logarithms. Can you compute them? (Hint: a few of the below might be undefined.)

19. What is $\ln(e^2)$?
20. What is $\log_3 \frac{1}{27}$?
21. What is $\log_2 0$?

22. What is $\log_{10} 1$?
23. What's the approximate value of $\log_2 e$? Is it a number less than 1? Between 1 and 2? Between 2 and 3? Bigger than 3?
24. What is $\log_5 \sqrt{5}$?
25. What is $\log_5(-5)$?
26. What is $\log_5 \frac{1}{5}$?
27. What is $\log_5 25$?
28. What is $\log_5(5^{\frac{3}{2}})$?

Do you know the rules of logarithms and how to use them to simplify expressions?

29. What is $\log_6(2) + \log_6(3)$?
30. Simplify $\ln 16 - 5 \ln 2 + \ln 1$.
31. True or false? $(\ln 3)^2 = \ln(3^2)$
32. True or false? $\frac{\ln x}{\ln y} = \ln x - \ln y$
33. Simplify $e^{\ln 17} - \log_2(4^{12})$.
34. What is $e^{4 \ln \pi}$?
35. What is $\ln(\frac{1}{e^2})^3$?

Can you solve equations that involve exponents and logarithms?

36. Solve $\log_7(x + 5) - \log_7(x - 1) = 2$.
37. Solve $\log_2(x^2) - \log_2(3x - 8) = 2$.
38. Solve $16^x = 45$.
39. Solve $10^{sinx} = 1$.
40. Solve $2 \ln(x + 1) - 1 = 0$.
41. Solve $2 \ln x = 4$.
42. Solve $e^{x^2+2x-3} = 1$.
43. Solve $\log_2(3 - x) = 3$.
44. Solve $3^{1-2x} = 27$.
45. Solve $\log_4(3x) = \frac{1}{2}$.
46. Solve $4 \cdot 16^{-3x} = 16^{3x-2}$.
47. Solve $e^{x-1} - 5 = 5$.
48. Solve $e^{x+1} = 5^{x+1}$.
49. Solve $e^{x+1} = 2 \cdot 3^{x-2}$.
50. Solve $2 \ln x = \ln(4x + 6) - \ln 2$
51. Solve $\ln(2x - 1) + \ln(3x - 2) = \ln 7$
52. Find a solution to $\log_{10}(\tan x) = 2$. (Challenge problem.)
53. Solve $e^{2x} + (e^x - 1)^2 = 1$.

54. Solve $(4\log_{10} x + 3) \cdot \log_{10} x^2 + 1 = 0$.

Can you graph logarithmic functions?

55. Graph $\ln x$ and $\log_2 x$.

56. What's the relationship between the graphs of $f(x) = \log_3 x$ and $g(x) = 3^x$?

57. Graph $g(x) = 4\ln(x-2)$. Label 4 points on this graph.

58. Graph $f(x) = \log_3(x) + 5$. Label 4 points on this graph. Does it have any asymptotes?

Finally, can you use exponential functions to model real-world situations? We have four examples of such situations in this class – population growth, decay of radioactive material, compound interest, and temperature. The first two situations can be modeled using a "non-shifted" exponential equation, $Q(t) = Q_0e^{kt}$. The last one is modeled using a "shifted" exponential equation, $Q(t) = Q_0e^{kt} + C$. Compound interest may be modeled differently, depending on if we are dealing with continuously compounded interest or compounding per period (e.g., monthly, semi-annually). To make things easier on you, we'll only consider continuously compounded interest, which also can be modeled by $Q(t) = Q_0e^{kt}$.

59. Say you invest \$100 into a bank account that earns 10% interest, compounded continuously. How long will it take for you to earn \$300 in interest?

60. Let's say you have a radioactive isotope with a half life of 100 years. If you start with a sample of 60 mg, how long will it be before there is just 10 mg left?

61. The population of the United States in 1900 was about 76 million. In the year 2000, it was about 282 million. Assuming that population growth in the United States fits an exponential curve (not necessarily a good assumption), what will the population be in the year 2050?

62. Take a potato at 20 degrees. Stick it in an oven at 300 degrees. After 10 minutes, the potato is 40 degrees. How long will it be before the potato reaches 200 degrees?

63. Say a couple has a child, and plans for that child to attend UConn in 18 years. They approximate that tuition will be around \$200,000 for four years by that time. Let's say they have a bank account that earns 5%, compounded continuously. How much do they have to deposit today in order to have \$200,000 in 18 years?

64. A radioactive substance decays by 10% in 20 years. How long will it take to decay by 20%?

65. Bacteria are growing. Eww, gross. Anyway, there are 1000 bacteria when they start, and they triple in population every 5 hours. How long until there are 10,000 bacteria?

66. Take a turkey at 180 degrees. Pull it out of the oven, into a room that is 70 degrees. After one hour, the turkey has cooled to 160 degrees. How long until it reaches 150 degrees?

67. A 50 mg sample of radioactive substance decayed to 10 mg in five years. What is the the half-life of the isotope?

68. A sheet of paper is approximately 0.1 mm thick. By folding it in half, you double its thickness. How many times would you need to fold the sheet of paper in half, so that it would be thick enough to reach the Moon? The distance from the Earth to the Moon is approximately 384,400,000,000 mm.